Cylindrical Domain Walls and Gravitational Waves

-Einstein Rosen wave emission from momentarily static initial configuration -

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A self-gravitating cylindrical domain wall is considered as an example of non-spherical wall to clarify the interaction between a domain wall and gravitational waves. We consider the time evolution from a momentarily static initial configuration within an infinitesimal time interval using the metric junction formalism. We found that the wall with a large initial radius radiates large amplitude of the gravitational waves and undergoes its large back reaction.

1 Introduction

Domain walls could be formed as a topological defect associated with the discrete symmetry breaking in the universe. It is thought that, in the treatment neglecting gravity, an oscillatory motion of the wall becomes to be a source of gravitational waves even in the first order with respect to its oscillation amplitude. If so, the energy loss rate can be estimated by the quadrupole formula for gravitational wave emission¹, since it seems that the radiation reaction can be negligible.

However, the recent investigations² show that there is no dynamical freedom in the domain wall coupled to gravitational field. In Ref.2, a spherically symmetric self-gravitating domain wall is considered as a background and the general relativistic perturbations are studied. It is found that the domain wall does not emit gravitational waves spontaneously by its free oscillation. However, it is not clear whether the result is caused by the spherical symmetry or not. Indeed, there is no local mode of gravitational waves in spherically symmetric spacetime which is used as a background in the soluble models.

In this talk, we consider the interaction between a domain wall and gravitational waves in a less symmetric model. We consider a self-gravitating thin wall with cylindrical symmetry as an example of non-spherical one. We use Israel's metric junction formalism³ to analyze the self-gravitating thin wall. So, one can easily see that the motion of wall directly leads to the gravitational wave emission.

2 Cylindrical Self-gravitating Domain Wall

Now, we consider the self-gravitating cylindrical domain wall. For simplicity, we assume that spacetime consists of two vacuum regions with cylindrical symmetry.

A cylindrically symmetric spacetime is described by Weyl canonical form

$$ds^{2} = e^{2(\gamma - \psi)}(-dt^{2} + dr^{2}) + e^{2\psi}dz^{2} + r^{2}e^{-2\psi}d\phi^{2}.$$
 (1)

From this metric, the Einstein equations can be reduced to

$$\partial_t^2 \psi - \frac{1}{r} \partial_r \left(r \partial_r \psi \right) = 0, \tag{2}$$

$$\partial_r \gamma = r \left((\partial_t \psi)^2 + (\partial_r \psi)^2 \right), \tag{3}$$

$$\partial_t \gamma = 2r(\partial_t \psi)(\partial_r \psi). \tag{4}$$

The wave solution ψ , which is well-known as Einstein Rosen wave (ERW), is regarded as a cylindrically symmetric mode of gravitational wave. On the other hand, γ corresponds to the gravitational potential of a cylindrically symmetric spacetime.

The junction conditions for the wall with the radius R are given by

$$\left(\ddot{R} - \dot{R}\dot{\psi} + R\dot{\psi}^2\right) \left(\frac{1}{X_+} - \frac{1}{X_-}\right) + R\left(\frac{\left(D_\perp\psi\right)_+^2}{X_+} - \frac{\left(D_\perp\psi\right)_-^2}{X_-}\right) = -2\lambda, \quad (5)$$

$$(D_{\perp}\psi)_{+} - (D_{\perp}\psi)_{-} = -\lambda, \tag{6}$$

$$X_{+} - X_{-} = -2\lambda R \tag{7}$$

where $X_{+} = \sqrt{\dot{R}^2 + e^{-2\gamma_{+}}}$ and $X_{-} = \sqrt{\dot{R}^2 + 1}$. In (5)-(7), $(D_{\perp}\psi)_{+}$ $((D_{\perp}\psi)_{-})$ is the derivative of ψ just outside (inside) the wall along the normal direction to the world hyper sheet of the wall, λ is the wall energy density, the dot denotes the derivative with respect to the proper time τ on the wall, and γ_{+} corresponds to the deficit angle just outside the wall. The condition (6) is the boundary condition for the ERW on the wall, (7) is the equation of the wall motion. Equation (5) is the first derivative of (7) with respect to τ .

As the simplest case, we consider the momentarily static initial configuration, i.e., $\dot{R} = 0$, $\partial_t \psi = \partial_t \gamma = 0$, with the initial wall radius R_i . From the regularity at the symmetric axis, the inside region of the wall should be momentarily Minkowski spacetime, i.e.,

$$\psi_{\mathcal{M}} = 0, \quad \gamma_{\mathcal{M}} = 0, \tag{8}$$

while from (6), outside region is described by

$$\psi_{\mathcal{M}_{+}} = -\kappa_{+} \ln \left(\frac{r}{R_{i}} \right), \qquad \gamma_{\mathcal{M}_{+}} = \gamma_{+} + \kappa_{+}^{2} \ln \left(\frac{r}{R_{i}} \right),$$
(9)

where $\kappa_{+} = \lambda R_{i}/(1-2\lambda R_{i})$, $\gamma_{+} = -\ln(1-2\lambda R_{i})$, and the variables with the suffix \mathcal{M}_{\pm} mean the values in the outside and inside regions of the wall, respectively. From (5), the initial configuration (8) and (9) give us the initial acceleration $\ddot{R}_{i} = -(2-3\lambda R_{i})/R_{i}$. The time evolution is given by solving the Einstein equations (2)-(4) and junction conditions (5)-(7). The solution within the infinitesimal time interval is given in the form⁴

$$\psi_{\mathcal{M}_{+}} = -\kappa_{+} \ln \frac{r}{R_{i}} + \frac{1}{2} \left(\frac{B}{r^{1/2}} \right) (\Delta u)^{2} + O((\Delta u)^{3}), \tag{10}$$

$$\psi_{\mathcal{M}_{-}} = -\frac{1}{2} \left(\frac{B}{r^{1/2}} \right) (\Delta v)^2 + O((\Delta v)^3),$$
 (11)

$$R(\tau) = R_i + \frac{1}{2}\ddot{R}_i(\Delta\tau)^2 + \frac{1}{3}\frac{(1 - 2\lambda R_i)^3 \kappa_+ B}{\lambda R_i \sqrt{R_i}}(\Delta\tau)^3 + O((\Delta\tau)^4).$$
 (12)

where Δu (Δv) is the infinitesimal retarded (advanced) time interval from the initial time and the initial amplitude of ERW, B, is given by

$$B = -\lambda \ddot{R}_i / \left(2(1 - 2\lambda R_i)^3 \sqrt{R_i} \right). \tag{13}$$

We can see the later behavior of the solution by solving (2)-(7) order by order.

From the solution (10)-(13), we can easily see that the initial amplitude B of ERW is determined by the initial acceleration \ddot{R}_i of the wall and, on the other hand, the motion of the wall is directly affected by the amplitude B. Equation (12) shows that the effect of back reaction appears in the order of $(\Delta \tau)^3$. Since \ddot{R}_i is negative initially and the coefficient of the order of $(\Delta \tau)^3$ in (12) is positive, the back reaction by wave emission reduces the wall's acceleration. Furthermore, we can see that if $R_i \ll 1/(2\lambda)$, the amplitude of ERW is small and its back reaction is small. Indeed, the back reaction does not become important until the wall collapse to zero radius in this case. However, if the initial radius becomes large as $R_i \to 1/(2\lambda)$, ERW is emitted with large amplitude and the back reaction is not negligible.

3 Summary

We have investigated the behavior of a self-gravitating cylindrical domain wall. Originally, the problem to solve this system corresponds to the radiation reaction problem and we must not neglect the emission of gravitational waves. Here, we have solved the time evolution of the system within an infinitesimal time interval from the momentarily static initial configuration using the metric junction formalism. First, we have found that the acceleration of the wall decreases by the back reaction via the emission of gravitational waves. Secondly, the wall with small initial radius emits gravitational waves with small amplitude and then its back reaction can be negligible. On the other hand, we have seen that the wall with large initial radius emits gravitational waves with large amplitude and the motion of the wall is altered considerably by its back reaction. Thus, it is important to take the interaction between the wall and gravitational waves into account.

References

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